{ B.Sc Part I (Physics Hons) }
Dr. Satyadeo Narayan Singh
S.B. College, Ara

Question: (a) What do you understand by time dilation? On the basis of Lorentz transformation discuss the variation of time with velocity according to special theory of relativity. Explain why does a moving ~~body~~ clock appears to run slow. Explain the term proper time and non proper time. Show that $v << c$ Lorentz transformation for time reduce to Galelian transformation.

(b) Describe an experiment to verify time dilation.

Ans:- (a) **Time dilation**:→

Suppose two observers O and O' in the inertial system S and S' respectively are at rest w.r. to each other. They synchoronise their respective clocks and they agree that the time interval between any two events as measured by their own clocks is the same. Let the clock in the system S' give signals at regular intervals and suppose the system S'' moves to the right along the x-axis with a university velocity V w.r. to S'. The observer O in the fram S' keeps his clock at a fixed point x_1 in his system and meaures the time interval T_0 that elapses between two events (say two signals) that occur at time t_1 and t_2 in his frame.

$$\therefore \; t_2 - t_1 = T_0 \qquad \text{--------- (1)}$$

Let the time registered by the observer O' in the inertial system S'' between the same two events be t_1' and t_2'. As S'' is moving to right with a uniform veloaty V w.r. to S''. According to Lorentz-transformation

$$t_1' = \frac{t_2 - \frac{V}{c^2}x_1}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$\& \quad t_2' = \frac{t_2 - \frac{V}{c^2}x_1}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$\therefore t_2' - t_1' = \frac{(t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The clock in the system S' remains fixed at point x_1

$$\therefore x_2 = x_1$$

Hence $t_2' - t_1' = \dfrac{t_2 - t_1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Thus the moving observer O' in the inertial system S'' measures the time interval $t_2' - t_1' = T$ between the same two events for which the observer O in the system S measures the time interval

$$t_2 - t_1 = T_0$$

$$\therefore T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Why ~~clock~~ moving clock appears to run slow :→

As discussed above, the moving observer O' in the ~~inl~~ inertial system S'' measures the time interval T between the same two events for which the observer O in the system S measures time interval T_0 and

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As v is less than c, $T > T_0$, Hence the observer S'' measures a longer interval of time between the two events with his clock at rest w.r. to him. In other words, a clock in the frame S appears to go slow to an observer in the frame S'' who is in motion w.r. to the frame S. This is known as time dilation. If there are two observers in relative motion w.r. to each other, each observer would find the other moving observer's clock to run slow. Thus the consequence of time dilation is reciprocal. As an example of the phenomenon of time dilation, suppose that the velocity of the observer

$$v = .98c \text{ then}$$

$\sqrt{1-\frac{v^2}{c^2}} = \sqrt{1-\frac{.98^2c^2}{c^2}} = 0.2 \quad \therefore T = \frac{T_0}{0.2} = 5T_0$ (3)

In other words, if the moving observer in the inertial frame s'' measures a time interval of 5sec between two events by his own clock, the interval between the same two events, as measured by him on the clock of the stationary observer in the inertial frame S, will be 1second and vice-versa.

Proper and non proper time:→ we have seen that an observer O in the inertial frame S measures a time interval $T_0 = t_2 - t_1$ between two events by a clock fixed in his frame at a position, say x_1, whereas the time interval between the same two events is measured as $T = t_2' - t_1' = \frac{t_2 - t_1}{\sqrt{1-\frac{v^2}{c^2}}}$ by the observers O' in the inertial frame s'' moving with velocity v in the +x direction w.r.to clock in frame s''.

The time interval between two events measured by an observer from an inertial frame which is moving w.r. to the clock is known as non-proper or relativistic time. when v<<c, where v<<c Lorentz transformation for time reduce to Galelian transformation.

(b) Experimental verification of time dilation:→

The high energy particles produced by a synchrocyclton give rise to π^+ mesons moving with a velocity 0.99c when they strike a target. To verify experimentally time dilation, as given by the theory of relativity, the flux of these π^+ measons was measured at two places 30 m apart. The π^+ measons decay with a half-life of $t = 1.8 \times 10^{-8}$ sec. The time taken by π^+ ~~mea~~ mesons to travel a distance of 30 meters as measured by an observer at rest in the laboratory is given by

$$T = \frac{30}{0.99 \times 3 \times 10^8} = 10 \times 10^{-8} \text{ sec (Approx)}$$

If No is the number of $\frac{30}{0.99 \times 3 \times 10^8} = 10 \times 10^{-8}$ measons in the begining and N after travelling a distance of 30 meters i,e after a time T.